Stability of Ultimate Motion in an Attitude Control System with Unbalanced Thrusters

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Theme

A SINGLE axis attitude control system, employing an integral pulse frequency modulator and thrusters having unsymmetrical force characteristics, is analyzed for the stability of its ultimate motion around a small equilibrium region, and for convergence of the system trajectory from large initial states to this region. A type of motion, which for practical purposes is equivalent to a two-pulse limit cycle, is defined and referred to here as "favorable motion." Relationships among the several system parameters are established and these ensure the existence of favorable motion. To establish the convergence from large initial states, a function similar to a Lyapunov function, but involving the concept of the minimum distance between a point and a set, is used as an analytical tool. The analytical results are displayed conveniently on a single graph and are verified by experiments on a computer simulation.

Contents

In 1963 Farrenkopf, Sabroff, and Wheeler investigated the limit cycle performance of a simple attitude control system, which was linear except for the integral pulse frequency (IPF) control moment actuator.[1] In 1969, Clark and Franklin extended this work by adding a nonlinear (deadzone) element in the control loop at the input to the IPF modulator. In this paper the previous work is extended by adding to the system of Clark and Franklin the additional feature of unbalanced control moment actuators. The impulse moment in one direction may differ from that in the other direction by any amount between zero and 50%. Small differences of this sort are bound to occur because of manufacturing tolerances of the valves, piping, and other hardware associated with the thrusters. Differences up to 50% could occur in the case where dual thrusters are used for reliability and one fails shut.

The IPF modulator is largely outdated as an attitude controller for spacecraft. Nevertheless, the analytical techniques developed here extend those developed in the 1963 and 1969 work and are applicable to other types of control systems which still employ IPF controllers. The results here may also be applied directly in the analysis of more modern types of attitude controllers. The pulse-width pulse-frequency modulator, for example, behaves like the IPF modulator in the region of the equilibrium point, which is where the ultimate state motion occurs.

The system which is depicted in Fig. 1 is characterized by the seven parameters J, K, a, θ_d, μ, A, and α. Except for α, these are the same parameters used by Clark and Franklin if the γ in their analysis is set to zero.[2,3] The strength of the impulse moment which increases $\dot{\theta}$ here is $+\mu$, and that which decreases $\dot{\theta}$ is $-\mu(1-\alpha)$ where α is a fixed parameter between 0 and 0.5 which represents the unbalanced thrusters. The dynamic state vector of this system may be projected onto a plane, called the q plane here, and adequately represented in that plane insofar as the ultimate motion of the system is concerned. The coordinate axes of that plane are z and y, so we regard q as the ordered pair (z,y)[4]

$$q=(z,y) \tag{1}$$

where the state variables z and y are related to θ and $\dot{\theta}$ as follows

$$y=\left(\frac{J}{\mu}\right)\dot{\theta}, \qquad z=y+\left(\frac{J}{\mu a}\right)\theta \tag{2}$$

It is convenient to introduce the following two coefficients:

$$\eta=\frac{\theta_d J}{\mu a}, \qquad c=\frac{2AJ}{K\mu a^2} \tag{3}$$

where A is the threshold level employed in the IPF modulator and is used differently here than in Ref. 2. In this paper we restrict η to the range $\eta \geq 0.5$.

Because of the nature of the IPF modulator, the state variable associated with its integrator may, for the purpose of trajectory calculations, be suppressed so that the dynamic equations of motion for this system may be reduced to a set of two nonlinear difference equations.[3,4] We assume that the control pulses are impulses which occur at time t_k where $k=0,1,2,3,\ldots$ Then we use $q(t_k^+)$ to identify the state of the system. The notation employed is

$$z_k=z(t_k^+), \quad y_R=k_k=y(t_k^+), \quad q_k=(z_k,y_k) \tag{4}$$

We have six characteristic regions in the q plane, so the matrix difference equation

$$q_{k+1}=F(q_k) \tag{5}$$

from which all of the subsequent analysis follows, must be written as six sets of equations, each set corresponding to one of the six regions. These six regions along with the regions T_o and the Target Zone, T_z and boundaries are illustrated in Fig. 2. Portions of trajectories starting in R_5 and R_2 are also shown.

The conditions are established under which a trajectory starting from an arbitrary point q_o outside the Target Zone will eventually have a switch point q_k within the Target Zone. A lengthy and complex analysis in Ref. 4 shows that if (c,α) lies anywhere in regions (f), (b), or (u), except for points along the right-hand edges of these regions in Fig. 3, and if $\eta \geq 0.5$, such convergence will occur. This analysis employs geometrical means to show that a trajectory, starting at an arbitrary q_o, will enter a prescribed region of the (y,z) plane within a finite time or that it will enter the Target Zone. This prescribed region is the union of R_4 and R_6 in Fig. 2. Having reduced the problem this far, the proof is completed by introducing a Lyapunov-like function, called the convergence function, and using it, with some further geometrical properties of the state transition equations. The convergence func-

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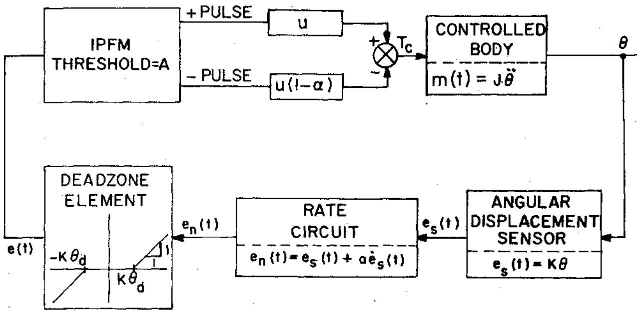

Fig. 1 Satellite attitude control system.

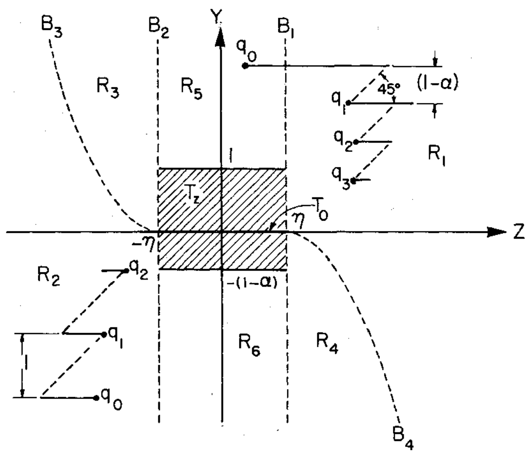

Fig. 2 Characteristic regions in the normalized phase plane.

tion is defined using the concept of the minimum distance d, between a point and a set.

It is not possible to study the ultimate motion of this system in terms of limit cycle oscillations. If α happens to be irrational, a limit cycle oscillation cannot exist. Even if α is rational, a limit cycle oscillation of, say, 50 pulses might occur even though the trajectory lies close to the origin and represents desirable behavior. Thus, we are led to the concept of "favorable motion" which for practical purposes (e.g., control fuel economy and pointing accuracy) is equivalent to two-pulse limit cycle motion (for small α) or three-pulse limit cycle motion (for α near 0.5).

Favorable Motion (FM) is defined as follows. If in the sequence of control pulses emitted at instants t_o, t_1, t_2,... (to which correspond the sequence of states q_o, q_1, q_2,...) there is an instant t_k (and a corresponding state q_k) such that the sub-sequence of control pulses subsequent to t_k contain not more than two successive negative pulses and no two successive positive pulses, then the system is said to exhibit Favorable Motion. Any other type of ultimate motion is said

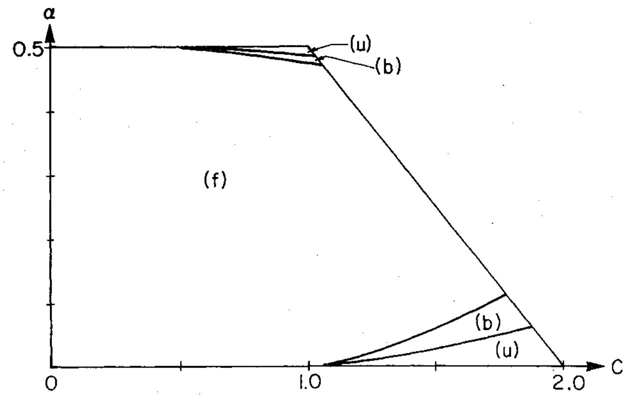

Fig. 3 $c-\alpha$ regions which define types of ultimate motion.

to be Unfavorable Motion. A two pulse limit cycle oscillation ($\alpha=0$), and a three pulse limit cycle oscillation ($\alpha=0.5$), exemplify favorable ultimate motion. On the other hand, if $\alpha=1/3$, and a limit cycle oscillation exists such that three negative pulses are followed by two positive pulses, then the ultimate motion is regarded as unfavorable.

To investigate the conditions in which favorable ultimate motion exists, we first assume that there exists a q_k lying in the Target Zone, T_z. The sufficient conditions are established by utilizing the geometric characteristics of the state transition equations to prove seven properties. A flow diagram showing all possible state transitions between the regions defined in Fig. 2 was constructed using these seven properties. Favorable ultimate motion was established by tracing the progress of all possible trajectories in this flow diagram, and was determined to occur for c and α lying in region f in Fig. 3, where the boundaries are carefully drawn to scale. Equations which define these boundaries are very complicated so α cannot be written as a closed form function of c. Using a similar procedure, it was established that unfavorable motion occurs in regions (u) and both favorable and unfavorable motion can occur in regions (b) in Fig. 3.

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